

Ques - Form an integral equation corresponding to the differential equation given by

$$\frac{d^2y}{dx^2} + xy = 1$$

$$y(0) = y'(0) = 0$$

Solⁿ

The given differential equation is

$$\frac{d^2y}{dx^2} + xy = 1 \quad \text{--- (1)}$$

with initial conditions

$$y(0) = y'(0) = 0 \quad \text{--- (2)}$$

Let us suppose

$$\frac{d^2y}{dx^2} = u(x) \quad \text{--- (3)}$$

Integrating equation (3) w.r. to x from 0 to x , we get

$$\left[\frac{dy}{dx} \right]_0^x = \int_0^x u(x) dx$$

$$\Rightarrow \frac{dy}{dx} - y'(0) = \int_0^x u(x) dx$$

$$\Rightarrow \frac{dy}{dx} = \int_0^x u(x) dx \quad \text{--- (4)}$$

$$\Rightarrow \frac{dy}{dx} = \int_0^x u(t) dt \quad \text{--- (5)} \quad [\because y'(0) = 0]$$

$$y(x) - y(0) = \int_0^x u(x) dx^2$$

$$\Rightarrow y(x) = \int_0^x (x-t) u(t) dt \quad \text{--- (6)} \quad [\because y(0) = 0]$$

Now putting the values of $\frac{d^2y}{dx^2}$ and y from above equation, in the given equation, we get

$$u(x) + x \int_0^x (x-t) u(t) dt = 1$$

$$\Rightarrow u(x) = 1 - \int_0^x x(x-t) u(t) dt \quad \text{--- (7)}$$

which is the required integral equation.

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